Two distinct positive integers a and b satisfy the equation

$$a^2 - b^2 = 2018 - 2a$$

What is the value of $a+b$?

We rearrange the equation:

$$a^2 + 2a - b^2 = 2018 \quad [+2a \text{ on both sides}]$$

$$\Rightarrow a^2 + 2a + 1 - b^2 = 2019 \quad [+1 \text{ on both sides}]$$

$$\Rightarrow (a+1)^2 - b^2 = 2019 \quad [\text{Factorise}]$$

$$\Rightarrow (a+1-b)(a+1+b) = 2019 \quad [\text{difference of squares}]$$

Now, since a and b are integers, $a+1-b$ and $a+1+b$ must also be integers. So we search for factors of 2019:

$$2019 \times 1$$
$$673 \times 3$$

Note that $a+1+b > a+1-b$ so in checking both cases we will assign it the larger of the factors.

$$\begin{cases} a+1-b = 1 \\ a+1+b = 2019 \end{cases}$$

$$\Rightarrow \begin{cases} a-b=0 \\ a+b=2018 \end{cases}$$

But then $a=b$, which is a contradiction (were told that a and b are different)

$$\begin{cases} a+1-b=3 \\ a+1+b=673 \end{cases}$$

$$\Rightarrow \begin{cases} a-b=2 \\ a+b=672 \end{cases}$$

So the answer is $\boxed{672}$