

A real number x satisfies

$$\sqrt{49-x^2} - \sqrt{25-x^2} = 3$$

What is the value of

$$\sqrt{49-x^2} + \sqrt{25-x^2}$$

For the sake of notational convenience, let us write

$$a := \sqrt{49-x^2}$$

$$b := \sqrt{25-x^2}$$

Now consider the difference of two squares:

$$a^2 - b^2 = (a-b)(a+b)$$

We have

$$\begin{aligned} a^2 - b^2 &= (\sqrt{49-x^2})^2 - (\sqrt{25-x^2})^2 \\ &= 49 - x^2 - (25 - x^2) \\ &= 24 \end{aligned}$$

and from the question we know
that $a - b = 3$.

So we have

$$a + b = \frac{a^2 - b^2}{a - b}$$

$$= \frac{24}{3}$$

$$= \boxed{8}$$