

What is the largest power of 2 that is a divisor of  $13^4 - 11^4$  ?

NO CALCULATORS!

Notice that we have a difference of squares:

$$\begin{aligned}13^4 - 11^4 &= (13^2)^2 - (11^2)^2 \\ &= (13^2 - 11^2)(13^2 + 11^2)\end{aligned}$$

We can apply the difference of squares formula again (to the first factor in the product):

$$\begin{aligned}&= (13 - 11)(13 + 11)(169 + 121) \\ &= 2 \times 24 \times 290\end{aligned}$$

Now rewrite the above product to emphasise all the 2's:

$$= 2 \times (2 \times 2 \times 2 \times 3) \times (2 \times 145)$$

There are five twos in the above

product, hence the largest power of 2  
which divides  $13^4 - 11^4$  is

$$2^5 = 32$$