How many ordered pairs (m,n) of positive integers with $m \geq n$ have the property that their squares differ by 96?

We have $m^2 - n^2 = 96$

$\Rightarrow (m-n)(m+n) = 2^5 \times 3$

There are $6 \times 2 = 12$ positive divisors of 96.

We consider cases:

$\begin{cases} m-n=1 \\ m+n=96 \end{cases}$

$\Rightarrow 2m = 97$

X

$\begin{cases} m-n=2 \\ m+n=48 \end{cases}$

$\Rightarrow 2m = 50$

$\Rightarrow m = 25$

$\Rightarrow n = 23$

$\begin{cases} m-n=3 \\ m+n=32 \end{cases}$

$\Rightarrow 2m = 35$

X

$\begin{cases} m-n=4 \\ m+n=24 \end{cases}$

$\Rightarrow 2m = 28$

$\Rightarrow m = 14$

$\Rightarrow n = 12$

$\begin{cases} m-n=6 \\ m+n=16 \end{cases}$

$\Rightarrow 2m = 22$

$\Rightarrow m = 11$

$\Rightarrow n = 9$

$\begin{cases} m-n=8 \\ m+n=12 \end{cases}$

$\Rightarrow 2m = 20$

$\Rightarrow m = 10$

$\Rightarrow n = 8$

So there are 4 pairs with this property.