

Solve For x : $4^x + 6^x = 9^x$

$$\Rightarrow \frac{4^x}{4^x} + \frac{6^x}{4^x} = \frac{9^x}{4^x}$$

$$\Rightarrow 1 + \left(\frac{6}{4}\right)^x = \left(\frac{9}{4}\right)^x$$

$$\begin{aligned}\Rightarrow 1 + \left(\frac{3}{2}\right)^x &= \left[\left(\frac{3}{2}\right)^2\right]^x \\ &= \left[\left(\frac{3}{2}\right)^x\right]^2\end{aligned}$$

Substitute $u = \left(\frac{3}{2}\right)^x$:

$$1 + u = u^2$$

Rearrange and solve using the quadratic Formula:

$$u^2 - u - 1 = 0$$

$$\Rightarrow u = \frac{1 \pm \sqrt{5}}{2}$$

But note that $u = \left(\frac{3}{2}\right)^x > 0$ so we only take the positive solution:

$$u = \frac{1 + \sqrt{5}}{2}$$

$$\Rightarrow \left(\frac{3}{2}\right)^x = \frac{1+\sqrt{5}}{2}$$

$$\Rightarrow x \ln\left(\frac{3}{2}\right) = \ln\left(\frac{1+\sqrt{5}}{2}\right)$$

$$\Rightarrow \boxed{x = \frac{\ln\left(\frac{1+\sqrt{5}}{2}\right)}{\ln\left(\frac{3}{2}\right)}}$$