Log Laws

$\log(xy) = \log(x) + \log(y)$

Proof Let $x = a^m$, $y = a^n$

We have $\log_a(xy) = \log_a(a^m a^n)$

$= \log_a(a^{m+n})$

$= m+n$

$= \log_a(x) + \log_a(y)$

index law

$\log\left(\frac{x}{y}\right) = \log(x) - \log(y)$

Proof Let $x = a^m$, $y = a^n$

We have $\log_a(x/y) = \log_a(a^m/a^n)$

$= \log_a(a^{m-n})$

$= m-n$

$= \log_a(x) - \log_a(y)$

index Law

$\log(x^n) = n\log(x)$

Proof Let $x = a^m$ (so $m = \log_a(x)$)

We have $\log_a(x^n) = \log_a[(a^m)^n]$

$= \log_a(a^{mn})$

$= mn$

index law

$= n\log_a(x)$

change of base