a, b, c are distinct one-digit numbers.
What is the maximum value of the sum of the roots of

$$(x-a)(x-b) + (x-b)(x-c) = 0$$

First we rearrange the LHS into a more standard form:

$$(x-a)(x-b) + (x-b)(x-c) = 0$$

$$\Rightarrow (x-b)[(x-a) + (x-c)] = 0$$

$$\Rightarrow (x-b)[2x - (a+c)] = 0$$

$$\Rightarrow 2x^2 - 2bx - (a+c)x + b(a+c) = 0$$

$$\Rightarrow 2x^2 + [-2b - a - c]x + b(a+c) = 0$$

The sum of roots is given by

$$\frac{-(-2b-a-c)}{2} = \frac{2b+a+c}{2}$$

$$= b + \frac{a}{2} + \frac{c}{2}$$

We need to maximise this quantity, keeping in mind that a, b, c are one-digit integers and distinct.

So we choose $b = 9$

$$a = 8 \quad \left[\text{or } a = 7\right.$$
$$c = 7 \quad \left. \phantom{\text{or }} c = 8\right]$$

So that the sum of the roots is

$$9 + \frac{8}{2} + \frac{7}{2} = \boxed{16.5}$$