Show that $b^2 + b + 1 = a^2$ has no positive integer solutions.

Rearrange : $b+1 = a^2 - b^2$

$= (a-b)(a+b)$ ⊛

Suppose that a positive integer solution exists.

Then $a^2 = b^2 + b + 1$

$> b^2$

$\Rightarrow \quad a > b$

So $a > b \geq 1$ and therefore

$a - b \geq 1$ (both integers)

$a + b \geq 2 + b$ (since $a \geq 2$)

Hence $(a-b)(a+b) \geq 1 \times (2+b) = 2+b$

Substitute ⊛ : $b+1 \geq b+2$

which is impossible.

So b^2+b+1 has no positive integer solutions.