There is a prime p s.t. $16p+1$ is the cube of a positive integer. Find p

$$16p + 1 = n^3$$

$$\Rightarrow \quad n^3 - 1 = 16p$$

$$\Rightarrow \quad (n-1)(n^2+n+1) = 16p$$

Now, either $(n-1) | p$ or $(n^2+n+1) | p$. But since p is a prime, we must have $n-1=p$ or $n^2+n+1=p$.

Note that $n-1=1$ and n^2+n+1 (the other factor of p) are both impossible since

$$n-1 = 1 \Rightarrow n = 2$$

$$\Rightarrow \underbrace{16p+1}_{\text{odd}} = \underbrace{2^3}_{\text{even}} \quad \times$$

contradiction

and $$n^2+n+1 = 1 \Rightarrow n = 0$$

$$\Rightarrow 16p+1 = 0 \quad \times$$

So we now analyse cases:

Case 1

$n-1 = p$ and $n^2+n+1 = 16$

$\Rightarrow n^2+n-15=0$

$\Rightarrow \Delta = 1-4(-15)$

$= 61$ ×

Impossible - n needs to be an integer

Case 2

$n-1 = 16$ and $n^2+n+1 = p$

$\Rightarrow n=17$ and so

$p = 289 + 17 + 1 = 307$