

$$A = 1 \times 2 + 3 \times 4 + 5 \times 6 + \dots + 37 \times 38 + 39$$

$$B = 1 + 2 \times 3 + 4 \times 5 + \dots + 36 \times 37 + 38 \times 39$$

Positive diff between A & B.

We have

$$A = (1 \times 2) + (3 \times 4) + (5 \times 6) + \dots + (37 \times 38) + 39$$

$$B = 1 + (2 \times 3) + (4 \times 5) + \dots + (36 \times 37) + (38 \times 39)$$

So

$$B - A = (38 \times 39) - 39 = 37 \times 39$$

$$+ (36 \times 37) - (37 \times 38) = -2 \times 37$$

+

⋮

$$+ (4 \times 5) - (5 \times 6) = -2 \times 5$$

$$+ (3 \times 2) - (3 \times 4) = -2 \times 3$$

$$+ 1 - (1 \times 2) = -1$$

$$= -1 + 37 \times 39 + \sum_{n=1}^{n=18} (-2) \times (2n+1)$$

3

$$2 \times 1 + 1$$

37

$$2 \times 18 + 1$$

$$= -1 + 37 \times 39 + (-2)(2) \sum_{n=1}^{18} n + (-2) \sum_{n=1}^{18} 1$$

$$= -1 + 37 \times 39 - 4 \times \frac{18 \times 19}{2} - 2 \times 18$$

$$= -1 + 1443 - 684 - 36$$

$$= \boxed{722}$$

A better grouping:

$$\begin{aligned} B-A &= (1-39) + 2 \times \overbrace{(3-1)}^2 + 4 \times \overbrace{(5-3)}^2 \\ &\quad + 6 \times \overbrace{(7-5)}^2 + \dots + 36 \times \overbrace{(37-35)}^2 \\ &\quad + 38 \times \overbrace{(39-37)}^2 \end{aligned}$$

$$= -38 + 4 \underbrace{(1 + 2 + \dots + 19)}_{\frac{20 \times 19}{2}}$$