We have

$x + 4 = (y-2)^2$ ——— ①

$y + 4 = (x-2)^2$ ——— ②

$x \neq y$

Find $x^2 + y^2$

We have

① $\Rightarrow y^2 - 4y + 4 = x + 4$

$\Rightarrow y^2 = x + 4y$

and ② $\Rightarrow x^2 - 4x + 4 = y + 4$

$\Rightarrow x^2 = y + 4x$

So ① + ② $\Rightarrow x^2 + y^2 = (x + 4y) + (y + 4x)$

$= 5x + 5y$

$= 5(x+y)$ (*)

and ① − ② $\Rightarrow x + 4 - y - 4 = (y-2)^2 - (x-2)^2$

$\Rightarrow x - y = y^2 - 4y + 4 - [x^2 - 4x + 4]$

$= y^2 - x^2 - 4y + 4x$

$\Rightarrow 0 = y^2 - x^2 - 3y + 3x$ $[-x + y]$

$\Rightarrow x^2 - y^2 = 3x - 3y$

$\Rightarrow \frac{x^2 - y^2}{x - y} = \frac{3x - 3y}{x - y}$

[know $x \neq y$, hence $x - y \neq 0$, so can legally divide through by it]

$\Rightarrow x + y = 3$

Now substitute this into (*):

$$x^2 + y^2 = 5(3) = \boxed{15}$$