IF $4b^2 + \frac{1}{b^2} = 2$ then what is

$$8b^3 + \frac{1}{b^3} \; ?$$

Recall the difference of cubes Formula:

$$x^3 - y^3 = (x-y)(x^2 + xy + y^2)$$

IF we let $z = (-y)$ then we obtain the sum of cubes Formula:

$$x^3 + z^3 = (x+z)(x^2 - xz + z^2)$$

In the question, if we let

$$x = 2b$$

$$z = \frac{1}{b}$$

then we Find that

$$8b^3 + \frac{1}{b^3} = (2b)^3 + \left(\frac{1}{b}\right)^3$$

$$= x^3 + z^3$$

$$= (x+z)(x^2 - xz + z^2)$$

$$= \left(2b + \frac{1}{b}\right)\left((2b)^2 - (2b)\cdot\frac{1}{b} + \frac{1}{b^2}\right)$$

$$= \left(2b + \frac{1}{b}\right)\left(4b^2 - 2 + \frac{1}{b^2}\right)$$

$$= \left(2b + \frac{1}{b}\right) \cdot 0$$
$$= 0$$

since $4b^2 + \frac{1}{b^2} = 2 \Rightarrow 4b^2 + \frac{1}{b^2} - 2 = 0$