The sum of 18 consecutive positive integers is a perfect square. What is the smallest possible value of this sum?

$$\sum_{n=a}^{n=a+17} n = m^2$$

$$\Rightarrow \sum_{n=1}^{n=a+17} n - \sum_{n=1}^{n=a-1} n = m^2$$

$$\Rightarrow \frac{(a+17)(a+18)}{2} - \frac{(a-1)a}{2} = m^2$$

$$\Rightarrow a^2 + 35a + 306 - (a^2 - a) = 2m^2$$

$$\Rightarrow 36a + 306 = 2m^2$$

$$\Rightarrow 18a + 153 = m^2$$

To minimise m^2, we need to minimise a (ie the starting point for the summation is as low as possible)

Try a few values :

$a=0 \Rightarrow m^2 = 153 \quad \times$

$a=1 \Rightarrow m^2 = 171 \quad \times$

$a=2 \Rightarrow m^2 = 189 \quad \times$

$a=3 \Rightarrow m^2 = 207 \quad \times$

$a=4 \Rightarrow m^2 = 225 = 15^2 \quad \checkmark$