The polynomial $f(x) = x^6 - 9x^2 - 6x$ has exactly 3 critical points. Find a parabola through them.

Critical points occur when $f'(x) = 0$

$$f'(x) = 6x^5 - 18x - 6$$

$$= 6(x^5 - 3x - 1)$$

We have

$$xf'(x) = 6(x^6 - 3x^2 - x)$$

So

$$\frac{xf'(x)}{6} = x^6 - 3x^2 - x$$

$$= x^6 - 9x^2 + 6x^2 - 6x + 5x$$

$$= (x^6 - 9x^2 - 6x) + 6x^2 + 5x$$

$$= f(x) + 6x^2 + 5x$$

$$\Rightarrow f(x) = \frac{xf'(x)}{6} - 6x^2 - 5x$$

IF $f'(x) = 0$ (ie at critical points) then

$$f(x) = -6x^2 - 5x$$

This is the parabola that we require.