$\frac{1}{7}$ as a decimal

Long division is the main method

Let's use "guess - and - check"

<u>First dp:</u>

We know $0.1 \times 7 = 0.7$ too small

$0.2 \times 7 = 1.4$ too big

So the First dp is a 1

<u>Second dp</u>:

we know 0.11

$\vdots$

$0.14 \times 7 = 0.98$] too small (<1)

$0.15 \times 7 = 1.05$ too big (>1)

So the second dp is a 4

<u>Third dp</u>: $0.142 \times 7 = 0.994$ too small

$0.143 \times 7 = 1.001$ too big

So the third dp is a 2

<u>Fourth dp</u> : $0.1428 \times 7 = 0.9996$ too small

$0.1429 \times 7 = 1.0003$ too big

So the Fourth dp is an 8

<u>Fifth dp</u>: $0.14285 \times 7 = 0.99995$ too small

$0.14286 \times 7 = 1.00002$ too big

So the Fifth dp is a 5

<u>Sixth dp</u> $0.142857 = 0.999999$ too small

$0.142858 = 1.000006$ too big

So the sixth dp is a 7.

<u>Seventh dp</u> $0.1428571 = 0.9999997$ too small

$0.1428572 = 1.0000004$ too big

So the seventh dp is a 1.

We have Found a repetition and can now stop.

We conclude that

$$\frac{1}{7} = 0.\overline{142857}$$

Simpler one: $\frac{5}{6}$

We have $0.8 \times 6 = 4.8$ too small (<5)

$0.9 \times 6 = 5.4$ too big (>5)

$\therefore$ 1st dp is an 8

We have $0.83 \times 6 = 4.98$ too small (<5)

$0.84 \times 6 = 5.04$ too big (>5)

$\therefore$ 2nd dp is a 3

We have $0.833 \times 6 = 4.998$ too small

$0.834 \times 6 = 5.004$ too big

$\therefore$ 3rd dp is a 3

We have reached a repetition so we can stop:

$$\frac{5}{6} = 0.8\dot{3}$$

Why do we know we can stop when we reach a repetition? Because we're working with <u>rational numbers</u> so we know it is always possible to find a repeating decimal. To be 100% certain, we should check our work using geometric series when we're done:

$$0.8\dot{3} = 0.8 + 3\sum_{k=2}^{\infty} \frac{1}{10^k}$$

$$= 0.8 + 3 \cdot \frac{\frac{1}{100}}{1 - \frac{1}{10}}$$

$$= 0.8 + 3 \cdot \frac{\frac{1}{100}}{\frac{9}{10}}$$

$$= 0.8 + 3 \cdot \frac{1}{90}$$

$$= \frac{8}{10} + \frac{1}{30}$$

$$= \frac{24}{30} + \frac{1}{30}$$

$$= \frac{25}{30}$$

$$= \frac{5}{6}$$

and similarly for $1/7$.

How does this method work?

If you want to convert $\frac{x}{y}$ to a decimal, perform the calculations

$$[\text{guess}] \times y$$

with guess being your current guesses (modifying the decimal place of interest)

Compare your results to x - the correct dp is the largest for which your product is less than x.