

Positive integers  $a$  and  $b$  are such that the graphs of  $y = ax + 5$  and  $y = 3x + b$  intersect the  $x$ -axis at the same point. What is the sum of all possible  $x$ -coordinates of these points of intersection?

Intersection with  $x$ -axis when  $y = 0$

$$ax + 5 = 0 \Rightarrow x = -\frac{5}{a}$$

$$3x + b = 0 \Rightarrow x = -\frac{b}{3}$$

$$\text{So } -\frac{5}{a} = -\frac{b}{3}$$

$$\Rightarrow ab = 15$$

Since  $a, b$  are positive integers, the only possibilities are

$a$	$b$
1	15
3	5

$$\begin{array}{c|c} 5 & 3 \\ 15 & 1 \end{array}$$

So the possible  $x$ -intercepts are

$$\begin{array}{c|c} b & -\frac{b}{3} \\ \hline 15 & -5 \\ 5 & -\frac{5}{3} \\ 3 & -1 \\ 1 & -\frac{1}{3} \end{array}$$

and hence the required sum is

$$\begin{aligned} & -5 + -\frac{5}{3} + -1 + -\frac{1}{3} \\ & = -6 + -\frac{6}{3} \\ & = -6 - 2 \\ & = \boxed{-8} \end{aligned}$$