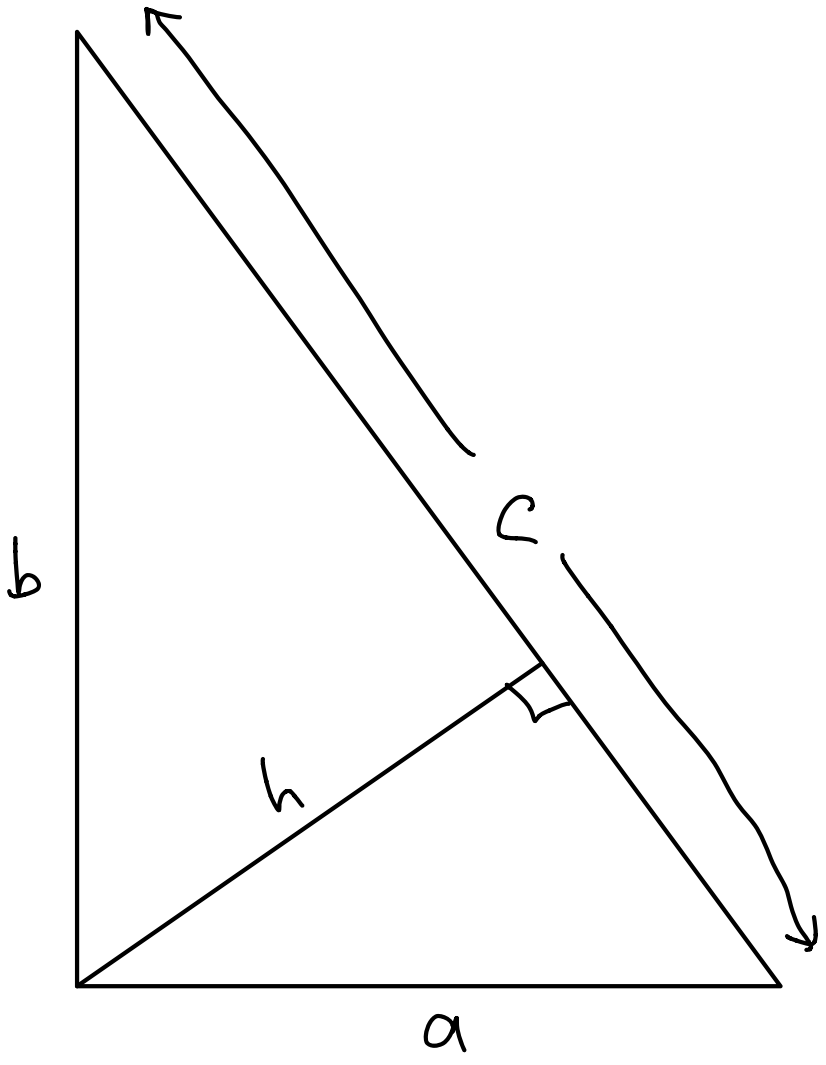
Inverse Pythagoras



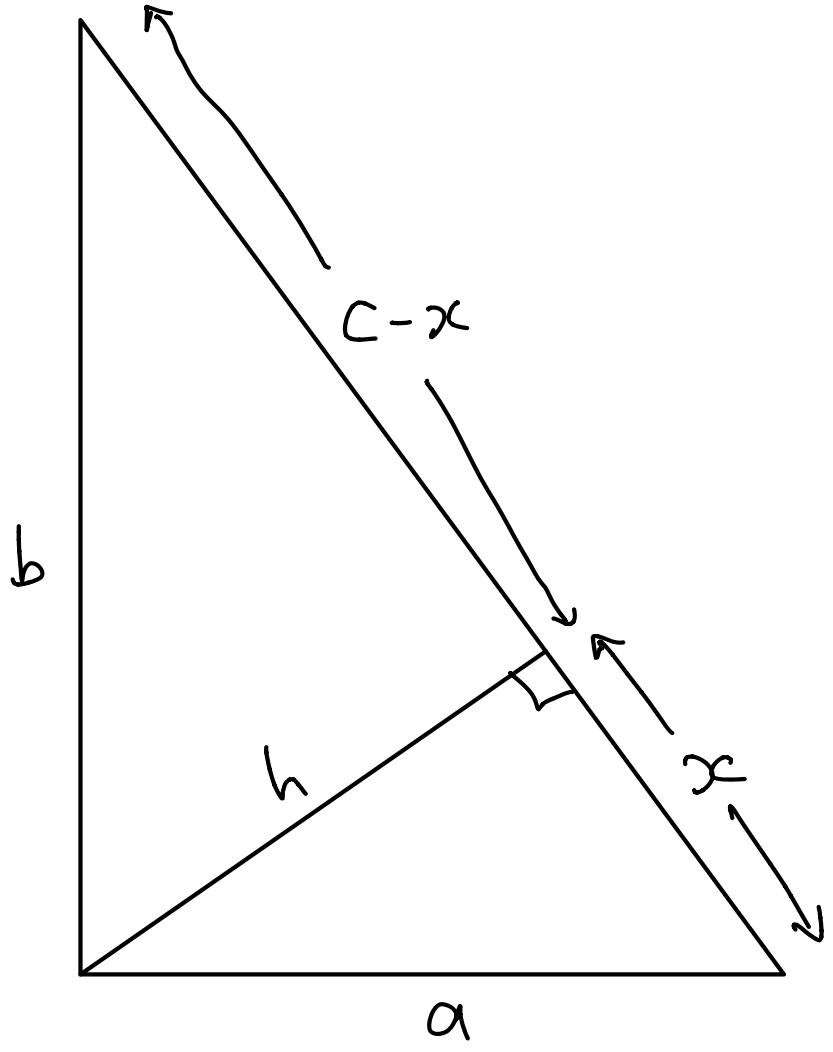
We know Pythagoras' Theorem, which says that for a right-angled triangle as above,

$$a^2 + b^2 = c^2$$

The Inverse Pythagoras Theorem states that for the triangle above, we have

$$\frac{1}{a^2} + \frac{1}{b^2} = \frac{1}{h^2}$$

Proof Let's draw another diagram:



We can apply Pythagoras' Theorem to the two small triangles:

(1) $x^2 + h^2 = a^2$

(2) $(c-x)^2 + h^2 = b^2$

however, this system is quite difficult to solve.

An easier way is to compare the area of the large triangle to the area of the

two smaller triangles.

We have

$$\text{Area of big } \triangle = \text{Area of lower } \triangle + \text{Area of upper } \triangle$$

$$\Rightarrow \frac{1}{2}ab = \frac{1}{2}xh + \frac{1}{2}(c-x)h$$

$$\Rightarrow ab = xh + ch - xh = ch$$

$$\Rightarrow c = \frac{ab}{h}$$

Substitute this into Pythagoras' theorem

$$a^2 + b^2 = c^2$$

$$= \left(\frac{ab}{h}\right)^2$$

$$= \frac{a^2b^2}{h^2}$$

Now divide through by a^2b^2:

$$\frac{1}{b^2} + \frac{1}{a^2} = \frac{1}{h^2}$$

as required.