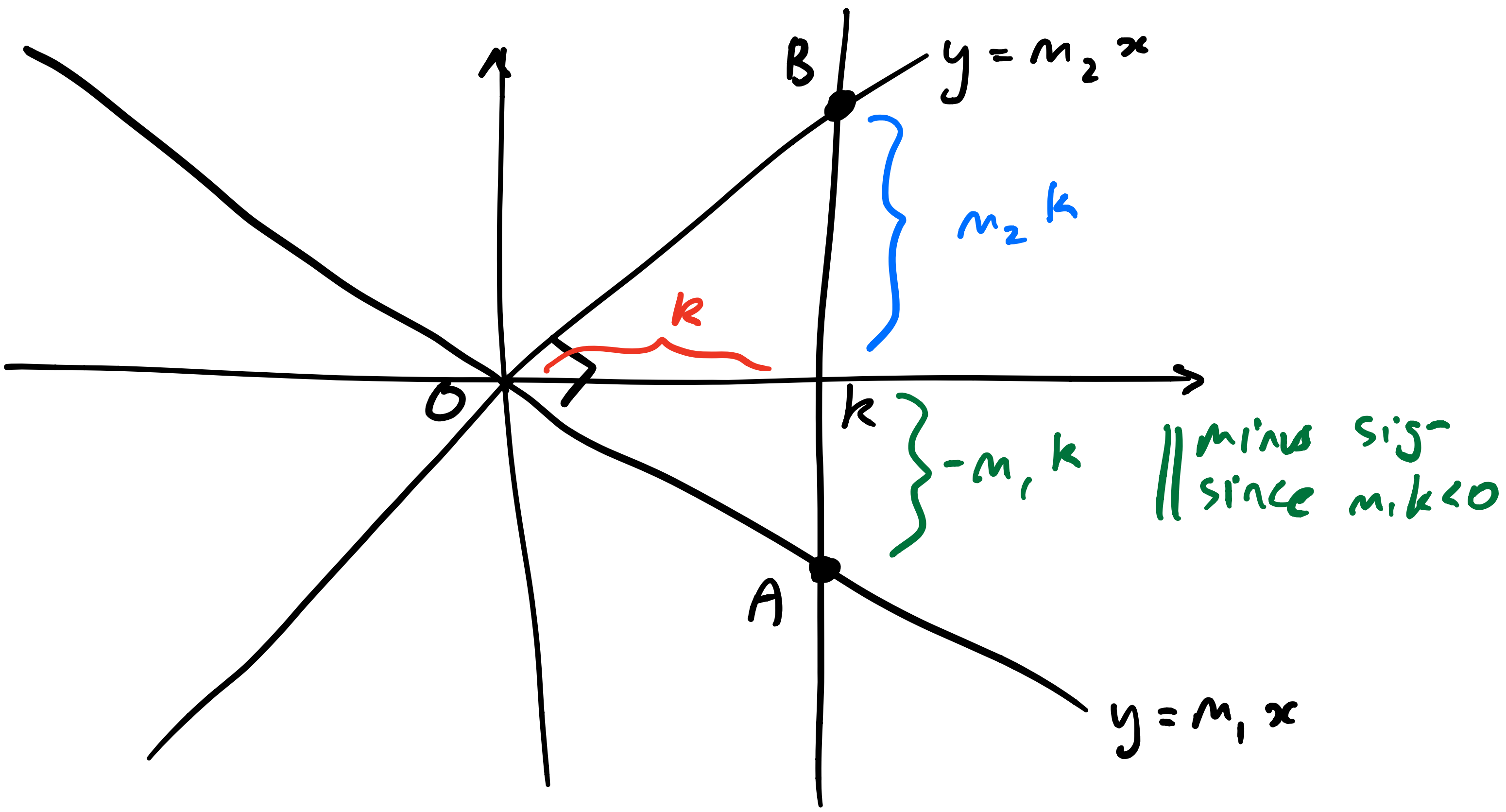
SOMEWHAT IMPRECISE → IMPROVE!!

Theorem $y = m_1 x + c_1$, $y = m_2 x + c_2$ } straight lines, $m_1, m_2 \neq 0$

If the two lines are perpendicular then $m_1 m_2 = -1$

Proof We can translate the lines so that their point of intersection is at the origin. They remain perpendicular. Their new eqns are $y = m_1 x$, $y = m_2 x$



Draw the line $x = k$, a positive real number

It meets our previous lines at A & B respectively.

Note that ΔOAB is right-angled, so we can apply Pythagoras' Theorem:

$$\overline{OA}^2 + \overline{OB}^2 = \overline{AB}^2$$

x-coord of A is k
y-coord of A is $m_1 k$

x-coord of B is k
y-coord of B is $m_2 k$

add the abs of y-coords from A & B to obtain vertical dist

$$\overline{AB}^2 = (m_2 k - m_1 k)^2$$

$\overline{OA}$: apply Pythagoras to small Δ below x-axis

$$k^2 + (m_1 k)^2 = \overline{OA}^2$$

$\overline{OB}$: apply Pythagoras to small Δ above x-axis

$$k^2 + (m_2 k)^2 = \overline{OB}^2$$

So we have

$$k^2 + (m_1 k)^2 + k^2 + (m_2 k)^2 = (m_2 k - m_1 k)^2$$

$$\Rightarrow k^2 + m_1^2 k^2 + k^2 + m_2^2 k^2 = m_2^2 k^2 + m_1^2 k^2 - 2m_1 m_2 k^2$$

$$\Rightarrow 2k^2 = -2m_1 m_2 k^2$$

$$\Rightarrow m_1 m_2 = -1$$

Q Prove that $y = 2x + 1$ and $y = -\frac{1}{2}x - 2$ are perpendicular

Q Prove that $5x - y + 10 = 0$ and

$$-\frac{1}{5}x - y + 100 = 0$$

are perpendicular

Quadratics

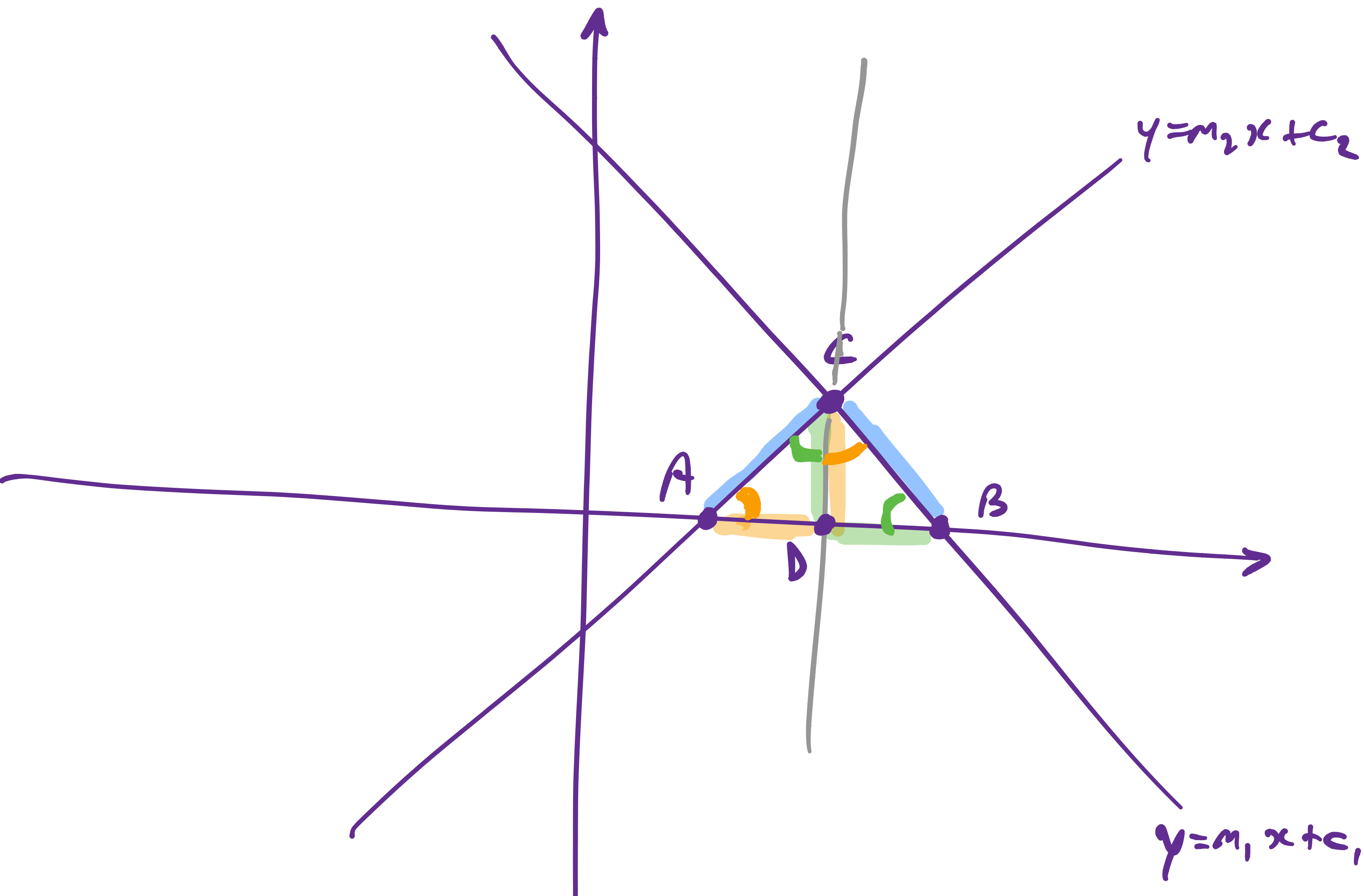
Q Solve $x^2 + 2x = -1$ for x

Q Graph $(x-3)(x+2) = 0$

Q Graph on the same set of axes

$$\begin{cases} y = x^2 \\ y = 2x^2 \\ y = \frac{1}{2}x^2 \end{cases} \qquad \begin{cases} y = x^2 \\ y = x^2 + 1 \\ y = x^2 - 1 \end{cases}$$

and identify roots



Suppose the two lines intersect at point C
Let A and B be the x-intercepts of the lines

Draw line parallel to y-axis passing through C
It intersects x-axis at D

Note that $\angle ACB = \angle ADC = \angle CDB = 90°$
Let $\angle CAD = \theta$. Then $\angle ACD = 90 - \theta$.
Since $\angle ACB$ is a right angle, we have $\angle DCB = \theta$. So $\angle CBD = 90 - \theta$
Hence, ΔADC is similar to ΔCDB (equiangular)

So corresponding sides are in proportion:

$$\frac{AD}{DC} = \frac{CD}{DB}$$

$$\Rightarrow AD \times DB = CD^2$$

Now, we use $\frac{\text{Rise}}{\text{Run}}$ to find the gradients of the lines using our triangles :

$$\frac{CD}{AD} = m_2$$

$$-\frac{CD}{DB} = m_1$$

So we have

$$m_1 m_2 = \frac{CD}{AD} \times \frac{-CD}{DB}$$

$$= \frac{-CD^2}{AD \times DB}$$

$$= \frac{-CD^2}{CD^2}$$

$$= -1$$