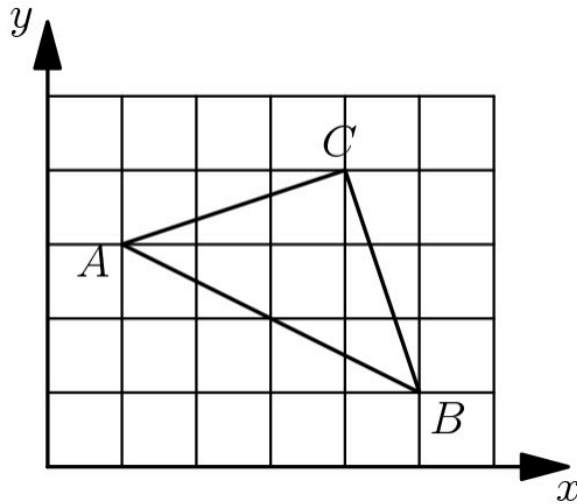


A triangle with vertices $A=(1,3)$
 $B=(5,1)$
 $C=(4,4)$

is plotted on a 6×5 grid. What fraction of the grid is covered by the triangle?



Firstly, note that the area of the whole grid is $6 \times 5 = 30$ units².

There are a few different approaches to this question, including Pick's Theorem [used to calculate areas of shapes on a grid] and Heron's Formula [used to find the area of a triangle based on side lengths without knowing perpendicular height]. We will approach the problem in a way which uses coordinate geometry and properties of lines.

Let us find the gradients of the lines

on which the sides AC and CB lie.

For AC $y = m_1x + b_1$

Sub in the given points:

$$\textcircled{1} \quad 3 = m_1 \cdot 1 + b_1$$

$$\textcircled{2} \quad 4 = m_1 \cdot 4 + b_1$$

Perform $\textcircled{2} - \textcircled{1}$: $1 = 3m_1 \Rightarrow m_1 = \frac{1}{3}$

For CB $y = m_2x + b_2$

Sub in the given points

$$\textcircled{1} \quad 4 = m_2 \cdot 4 + b_2$$

$$\textcircled{2} \quad 1 = m_2 \cdot 5 + b_2$$

Perform $\textcircled{2} - \textcircled{1}$: $-3 = m_2$

Now, note that we have $m_1 \times m_2 = -1$, hence the two lines are perpendicular and so $\angle ACB$ is a right angle.

Let us now compute the side lengths

AC and CB.

We have

$$AC = \sqrt{(4-3)^2 + (4-1)^2}$$

$$= \sqrt{1+9}$$

$$= \sqrt{10}$$

and

$$CB = \sqrt{(4-1)^2 + (4-5)^2}$$

$$= \sqrt{9+1}$$

$$= \sqrt{10}$$

So we have

$$\text{Area}(\triangle ABC) = \frac{1}{2} \times \text{Base} \times \text{Height}$$

$$= \frac{1}{2} \times \sqrt{10} \times \sqrt{10}$$

$$= \frac{10}{2}$$

$$= 5$$

Hence, the fraction of the grid covered by the triangle is

$$\frac{5}{30} = \boxed{\frac{1}{6}}$$