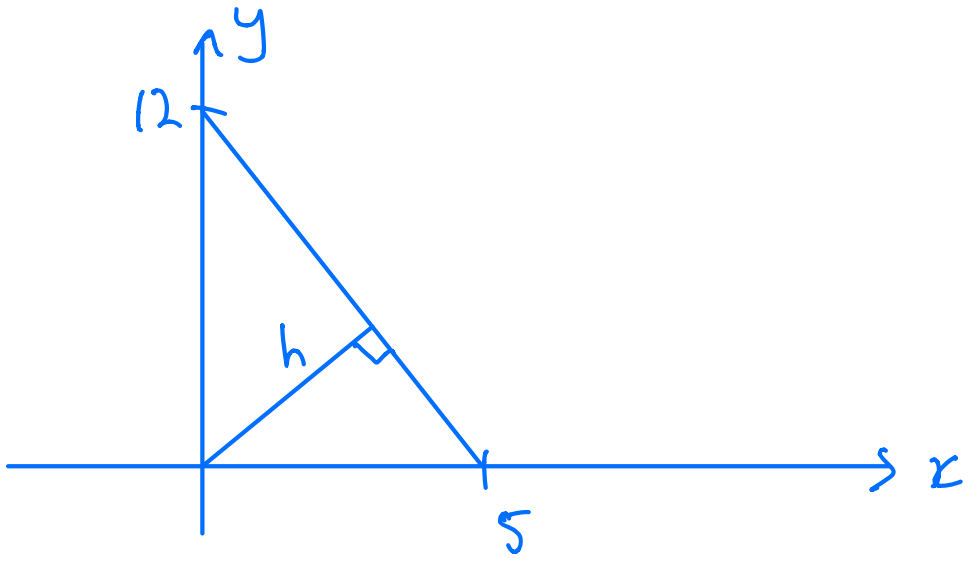
The line $12x + 5y = 60$ forms a triangle with the coordinate axes. What is the sum of the lengths of the altitudes of this triangle?

First draw a graph:



Intercepts: $x = 0 \Rightarrow 5y = 60$

$\Rightarrow y = 12$

$y = 0 \Rightarrow 12x = 60$

$\Rightarrow x = 5$

Two of the altitudes are therefore easy, 12 & 5.

For the third, we can use the Inverse Pythagorean Theorem:

$$\frac{1}{h^2} = \frac{1}{5^2} + \frac{1}{12^2}$$

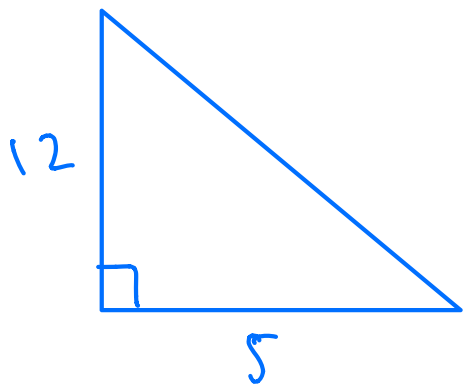
$$\Rightarrow \quad \frac{1}{h^2} = \frac{169}{3600}$$

$$\Rightarrow \quad h^2 = \frac{3600}{169}$$

$$\Rightarrow \quad h = \frac{60}{13}$$

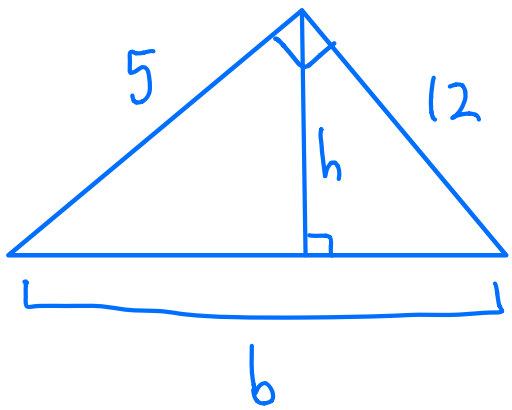
So the sum of the altitudes is $\frac{281}{13}$

Another way to do the problem (ie without Inverse Pythag) is to consider the area of the triangle.



Area = $\frac{1}{2}$ x base x height

= 30



By Pythagoras,

$b^2 = 5^2 + 12^2$

$\Rightarrow b = 13$

So Area $= \frac{1}{2} \times$ base $\times$ height

$= \frac{1}{2} \times 13 \times h$

But From the previous diagram, we know that Area $= 30$. Substitute in:

$$30 = \frac{13h}{2}$$

$$\Rightarrow 60 = 13h$$

$$\Rightarrow h = \frac{60}{13}$$

So the sum of the altitudes is

$$12 + 5 + \frac{60}{13} = \frac{281}{13}$$