Prove that $5^{2n+1} + 11^{2n+1} + 17^{2n+1}$ is divisible by 33 for every non-negative integer n.

$$33 = 3 \times 11$$

$$5 \equiv 2$$
$$5^2 \equiv 1$$
$$5^3 \equiv 2$$
$$\vdots$$

$$5^{2n+1} \equiv 2 \mod 3$$

$$11^{2n+1} \equiv 2 \mod 3$$

$$17^{2n+1} \equiv 2 \mod 3$$

$$17 \equiv 2$$
$$17^2 \equiv 1$$

$$11 \equiv 2$$
$$11^2 \equiv 1$$
$$11^3 \equiv 2$$

So

$$5^{2n+1} + 11^{2n+1} + 17^{2n+1} \equiv 2 + 2 + 2$$
$$\equiv 6$$
$$\equiv 0 \mod 3$$

$\therefore$ divisible by 3

$$11^{2n+1} \equiv 0 \mod 11$$

Now $5^{2n+1} = (11 - 6)^{2n+1}$

$$\equiv (-6)^{2n+1} \mod 11$$

and $17^{2n+1} = (11+6)^{2n+1}$

$$\equiv 6^{2n+1} \mod 11$$

$\therefore\ 5^{2n+1} + 11^{2n+1} + 17^{2n+1} \equiv (-6)^{2n+1} + 0 + 6^{2n+1} \mod 11$

$$\equiv -6^{2n+1} + 6^{2n+1} \mod 11$$

$$\equiv 0 \mod 11$$

$\therefore$ divisible by 11

Since the number is divisible by both 3 and 11, it is also divisible by 33.