Prove that 10201 is composite in any base

$$10201_n = 1\times 1 + 0\times n + 2\times n^2 + 0\times n^3 + 1\times n^4$$

$$= 1 + 2n^2 + n^4$$

$$= (n^2+1)^2 \quad \text{is a composite}$$

Since $n>1 \Rightarrow n^2+1>1$

Prove that 10101 is composite in any base

$$10101_n = 1\times 1 + 0\times n + 1\times n^2 + 0\times n^3 + 1\times n^4$$

$$= 1 + n^2 + n^4$$

$$= 1 + n^2 + n^4 + (n^2 - n^2) \quad \text{[complete the square]}$$

$$= 1 + 2n^2 + n^4 - n^2$$

$$= (n^2+1)^2 - n^2$$

$$= (n^2+1-n)(n^2+1+n)$$

is a composite (both Factors > 1)

1) Expand $(x^2+x+1)(x^3-x^2+1)$

$$(x^2+x+1)(x^3-x^2+1)$$

$$= x^5 + x^4 + x^3 - x^4 - x^3 - x^2$$

$$+ x^2 + x + 1$$

2) Prove that 100011 is composite in any base

$$100011_n = 1 \times 1 + 1 \times n + 0 \times n^2$$

$$+0\times n^3$$
$$+0\times n^4$$
$$+1\times n^5$$
$$= n^5 + n + 1$$
$$= (n^2+n+1)(n^3-n^2+1)$$

Composite since both factors >1

Factorise $f(x) = x^6 - a^2x^4 + 2x^3 + 1$

$$f(x) = x^6 - a^2x^4 + 2x^3 + 1$$
$$= x^6 + 2x^3 + 1 - a^2x^4$$
$$= (x^3+1)^2 - (ax^2)^2$$
$$= [x^3 + 1 - ax^2][x^3 + 1 + ax^2]$$