$$\frac{1}{1\cdot 2}+\frac{1}{2\cdot 3}+\frac{1}{3\cdot 4}+\dots+\frac{1}{99\cdot 100}$$

① $\frac{1}{2}$

② $\frac{1}{2}+\frac{1}{6}=\frac{6+2}{12}=\frac{8}{12}=\frac{2}{3}$

③ $\frac{1}{2}+\frac{1}{6}+\frac{1}{12}=\frac{2}{3}+\frac{1}{12}=\frac{8}{12}+\frac{1}{12}=\frac{9}{12}=\frac{3}{4}$

Guess: $\displaystyle\sum_{n=1}^{n=k}\frac{1}{n(n+1)}=\frac{k}{k+1}$

Induction: Suppose true for some $k\in\mathbb{Z}$

Then $\displaystyle\sum_{n=1}^{n=k+1}\frac{1}{n(n+1)}=\sum_{n=1}^{n=k}\frac{1}{n(n+1)}+\frac{1}{(k+1)(k+2)}$

$$=\frac{k}{k+1}+\frac{1}{(k+1)(k+2)}$$

$$=\frac{k(k+2)+1}{(k+1)(k+2)}$$

$$= \frac{k^2 + 2k + 1}{(k+1)(k+2)}$$

$$= \frac{(k+1)^2}{(k+1)(k+2)}$$

$$= \frac{k+1}{k+2}$$

$\therefore$ True by mathematical induction.

When $k = 99$: Sum $= \frac{99}{100}$