If $2\log(x-2y) = \log(x) + \log(y)$, Find $\frac{x}{y}$

$$\log[(x-2y)^2] = \log[xy]$$

$$\Rightarrow x^2 - 4xy + 4y^2 = xy$$

$$\Rightarrow x^2 - 5xy + 4y^2 = 0$$

Consider as quadratic in x:

$$x = \frac{5y \pm \sqrt{25y^2 - 4(4y^2)}}{2}$$

$$= \frac{5y \pm \sqrt{9y^2}}{2}$$

$$= \frac{5y \pm 3y}{2}$$

$$= \frac{2y}{2}, \frac{8y}{2}$$

$$= y, 4y$$

Then $\frac{x}{y} = 1$ or $\frac{x}{y} = 4$

Case 1 $x = y$

LHS $= 2\log(x - 2x) = 2\log(-x)$

RHS $= \log(x) + \log(x) = 2\log(x)$ ✗

Case 2 $x = 4y$

LHS $= 2\log(4y - 2y) = 2\log(2y) = \log[(2y)^2]$

$= \log(4y^2)$

RHS $= \log(4y) + \log(y) = \log(4y \times y) = \log(4y^2)$

$\therefore \frac{x}{y} = 4$ ✓

What went wrong with Case 1? Let's think about the domain of the log function. We need a positive argument. So x, y and $x - 2y$ are all positive. If $x = y$ then

$$x - 2y = -x < 0$$ Problem!

So $x = 4y$ was the only viable solution.