What is the value of a for which

$$\frac{1}{\log_2(a)} + \frac{1}{\log_3(a)} + \frac{1}{\log_4(a)} = 1 \quad ?$$

Change of base formula:

$$\log_x(a) = \frac{\log_b(a)}{\log_b(x)}$$

So

$$\log_4(a) = \frac{\log_2(a)}{\log_2(4)} = \frac{\log_2(a)}{2}$$

$$\log_3(a) = \frac{\log_2(a)}{\log_2(3)}$$

Hence

$$LHS = \frac{1}{\log_2(a)} + \frac{\log_2(3)}{\log_2(a)} + \frac{2}{\log_2(a)}$$

$$= \frac{3 + \log_2(3)}{\log_2(a)}$$

So $3 + \log_2(3) = \log_2(a)$

$\Rightarrow \quad 2^{3+\log_2(3)} = 2^{\log_2(a)}$

$\Rightarrow \quad 2^3 \, 2^{\log_2(3)} = a$

$\Rightarrow \quad a = 8 \times 3 = 24$

IF $a, b > 0$ and

$$\log_{ab}\left(a^{\frac{1}{3}} b^{-\frac{1}{2}}\right) = \frac{5k-3}{6} \qquad (k \neq 0)$$

Find $\log_a(b)$

$\log_a(x) = y$ means $a^y = x$

$$(ab)^{\frac{5k-3}{6}} = a^{\frac{1}{3}} b^{-\frac{1}{2}}$$

$\rightarrow a^{\frac{5k-3}{6} - \frac{1}{3}} \, b^{\frac{5k-3}{6} + \frac{1}{2}} = 1$

$\Rightarrow a^{\frac{5k-5}{6}} \, b^{\frac{5k}{6}} = 1$

$$\Rightarrow a^{5k-5} b^{5k} = 1$$

$$\Rightarrow a^{5k-5} = b^{-5k}$$

$$\Rightarrow \left(a^{5k-5}\right)^{-\frac{1}{5k}} = b$$

$$\Rightarrow a^{\frac{-5k+5}{5k}} = b$$

$$\Rightarrow a^{-1+\frac{1}{k}} = b$$

$$\Rightarrow \log_a(b) = -1 + \frac{1}{k}$$

$$= \frac{-k+1}{k}$$

$$= \frac{1-k}{k}$$