

Is it possible to write 1 as the sum of an even number of the reciprocals of odd integers?

Suppose by way of contradiction that

$$1 = \frac{1}{n_1} + \dots + \frac{1}{n_k}$$

where  $k$  even &  $n_1, \dots, n_k$  odd

Then

$$1 = \frac{n_2 \dots n_k + \dots + n_1 \dots n_{k-1}}{n_1 \dots n_k}$$

$$\Rightarrow n_1 \dots n_k = n_2 \dots n_k + \dots + n_1 \dots n_{k-1}$$

The LHS is a product of odd integers therefore also odd.

The RHS is the sum of an even

number of odd integers and therefore even

This is a contradiction, hence it is not possible.