

Find all number triples (x, y, z) s.t. when any one of these numbers is added to the product of the other two, the result is 2

$$\begin{cases} x + yz = 2 & \textcircled{1} \\ y + xz = 2 & \textcircled{2} \\ z + xy = 2 & \textcircled{3} \end{cases}$$

By symmetry, $x = y = z$

Sub into $\textcircled{1}$: (or any eqn)

$$x + x^2 = 2$$

$$\Rightarrow x^2 + x - 2 = 0$$

$$\begin{aligned} \Rightarrow x &= \frac{-1 \pm \sqrt{1 - 4(-2)}}{2} \\ &= \frac{-1 \pm \sqrt{1+8}}{2} \\ &= -1 \pm 3 \end{aligned}$$

$$\frac{2}{2}$$

$$= -\frac{4}{2}, \frac{2}{2}$$

$$= -2, 1$$

So the only solutions are

$$(-2, -2, -2)$$

and $(1, 1, 1)$

Check $1 + 1 \times 1 = 1 + 1 = 2$

$$-2 + (-2) \times (-2) = -2 + 4 = 2$$

