Prove that the product of 4 consecutive natural numbers cannot be a perfect square.

Product of four consecutive numbers $= f(n) = n(n+1)(n+2)(n+3)$

Try some values:

n	1	2	3	4
$f(n)$	24	120	360	840

Notice: $f(n)$ = square number -1

$f(n)+1 = n(n+1)(n+2)(n+3)+1$

$= [n(n+3)][(n+1)(n+2)]+1$

$= [n^2+3n][n^2+3n+2]+1$

$= [(n^2+3n+1)-1][(n^2+3n+1)+1]+1$

$= (n^2+3n+1)^2-1+1$

$= (n^2+3n+1)^2$

$\therefore f(n) = (n^2+3n+1)^2-1$

so it cannot be a perfect square