

If $\alpha \in (90^\circ, 180^\circ)$ simplify

$$\sqrt{\frac{1 - \sin \alpha}{1 + \sin \alpha}} - \sqrt{\frac{1 + \sin \alpha}{1 - \sin \alpha}}$$

$$\begin{aligned} \sqrt{\frac{1 - \sin \alpha}{1 + \sin \alpha}} - \sqrt{\frac{1 + \sin \alpha}{1 - \sin \alpha}} &= \sqrt{\frac{(1 - \sin \alpha)(1 - \sin \alpha)}{(1 + \sin \alpha)(1 - \sin \alpha)}} \\ &\quad - \sqrt{\frac{(1 + \sin \alpha)(1 + \sin \alpha)}{(1 - \sin \alpha)(1 + \sin \alpha)}} \end{aligned}$$

$$= \sqrt{\frac{(1 - \sin \alpha)^2}{1 - \sin^2 \alpha}} - \sqrt{\frac{(1 + \sin \alpha)^2}{1 - \sin^2 \alpha}}$$

$$= \sqrt{\frac{(1 - \sin \alpha)^2}{\cos^2 \alpha}} - \sqrt{\frac{(1 + \sin \alpha)^2}{\cos^2 \alpha}}$$

$$= \frac{1 - \sin \alpha}{-\cos \alpha} - \frac{1 + \sin \alpha}{-\cos \alpha} \quad (*)$$

$$= \frac{1 - \sin \alpha - 1 - \sin \alpha}{-\cos \alpha}$$

$$= \frac{-2 \sin \alpha}{-\cos \alpha}$$

$$= \tan \alpha$$

⊛ For $\alpha \in (90^\circ; 180^\circ)$ we have

$$\sqrt{\cos^2 \alpha} = |\cos \alpha| = -\cos \alpha$$

Since \cos is negative in the 2nd quadrant