

Solve

$$3 \cdot 4^{\sqrt{x}-1} - 2 \cdot 6^{\sqrt{x}-1} - 9^{\sqrt{x}-1} > 0$$

Let  $y = \sqrt{x} - 1$

So

$$3 \cdot 4^y - 2 \cdot 6^y - 9^y > 0$$

Divide through by  $9^y$ :

$$\frac{3 \cdot 4^y}{9^y} - \frac{2 \cdot 6^y}{9^y} - 1 > 0$$

$$\Rightarrow 3 \cdot \left(\frac{2}{3}\right)^{2y} - 2 \cdot \left(\frac{2}{3}\right)^y - 1 > 0$$

Let  $w = \left(\frac{2}{3}\right)^y$

$$\Rightarrow 3 \cdot w^2 - 2 \cdot w - 1 > 0$$

This is a quadratic

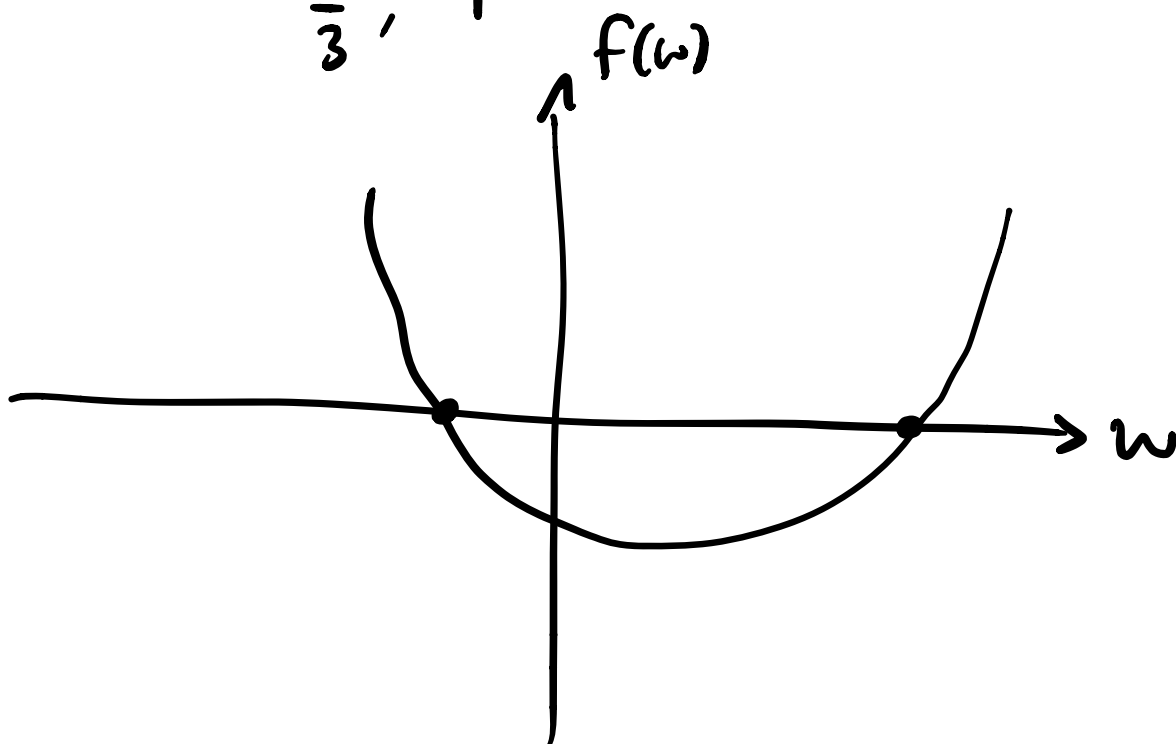
$$w = \frac{2 \pm \sqrt{4 - 4 \times 3 \times (-1)}}{2 \times 3}$$

$$2 \times 3$$

$$= \frac{2 \pm 4}{6}$$

$$= -\frac{2}{6}, \frac{6}{6}$$

$$= -\frac{1}{3}, 1$$



So  $f(w) > 0$  for  $w < -\frac{1}{3}$  &  $w > 1$

i.e.  $\left(\frac{2}{3}\right)^{\sqrt{x}-1} < -\frac{1}{3}$  &  $\left(\frac{2}{3}\right)^{\sqrt{x}-1} > 1$

But  $\left(\frac{2}{3}\right)^{\sqrt{x}-1} > 0$  and so the only

possible solutions are for

$$\left(\frac{2}{3}\right)^{\sqrt{x}-1} > 1$$

$$\Rightarrow \sqrt{x} - 1 < 0$$

$$\Rightarrow \sqrt{x} < 1$$

$$\Rightarrow x < 1$$

$$\text{So } x \in [0, 1)$$