Find the sum of all positive integers $b<1000$ s.t. 36_b is a perfect square and 27_b is a perfect cube

$$36_b = 3b + 6$$

$$27_b = 2b + 7$$

$3b+6$ perfect square

$= 3(b+2)$

$\Rightarrow b+2 = 3n^2$ for $n \in \mathbb{N}$

Now $b<1000 \Rightarrow 3n^2 - 2 < 1000$

$\Rightarrow 3n^2 < 1002$

$\Rightarrow n^2 < 334$

$\Rightarrow n \leq 18$

Check second condition by cases:

n	$b = 3n^2 - 2$	$2b+7$

1	$3-2=1$	9	x
2	$12-2=10$	27	✓
3	$27-2=25$	57	x
4	$48-2=46$	99	x
5	$75-2=73$	153	x
6	$108-2=106$	219	x
7	$147-2=145$	297	x
8	190	387	x
9	241	489	x
10	298	603	x
11	361	729	✓
12	430	867	x
13	505	1017	x
14	586	1179	x
15	675	1357	x
16	766	1539	x
17	865	1737	x
18	970	1947	x

So the required sum is

$$10 + 361 = 371$$