

The number n can be written in base 14 as abc , in base 15 as acb and in base 6 as $acac$ ($a > 0$). Find the base 10 representation of n .

$$\begin{aligned} n &= c \times 1 + b \times 14 + a \times 14^2 \\ &= c + 14b + 196a \end{aligned} \quad \left. \begin{array}{l} \text{base} \\ 14 \end{array} \right\} \quad (1)$$

$$\begin{aligned} n &= b \times 1 + c \times 15 + a \times 15^2 \\ &= b + 15c + 225a \end{aligned} \quad \left. \begin{array}{l} \text{base} \\ 15 \end{array} \right\} \quad (2)$$

$$\begin{aligned} n &= c \times 1 + a \times 6 + c \times 36 + a \times 216 \\ &= 37c + 222a \end{aligned} \quad \left. \begin{array}{l} \text{base} \\ 6 \end{array} \right\} \quad (3)$$

Know $a \in \{1, \dots, 5\}$ • leading digit
 • involved in base 6 rep
 $b \in \{0, \dots, 13\}$ • involved in base 14 rep
 $c \in \{0, \dots, 5\}$ • involved in base 6 rep

From (2) and (3):

$$b + 15c + 225a = 37c + 222a$$

$$\Rightarrow b = 22c - 3a$$

From ① and ② :

$$c + 14b + 196a = b + 15c + 225a$$

$$\Rightarrow 13b = 14c + 29a$$

$$\Rightarrow 13(22c - 3a) = 14c + 29a$$

$$\Rightarrow 286c - 39a = 14c + 29a$$

$$\Rightarrow 272c = 68a$$

$$\Rightarrow 136c = 34a$$

$$\Rightarrow 68c = 17a$$

$$\Rightarrow 4c = a$$

Given that $1 \leq a \leq 5$ and $0 \leq c \leq 5$, we

must have $c = 1$ and $a = 4$

$$\text{Hence } n = 37c + 222a$$

$$= 37 + 888$$

$$= 925$$

$$\text{Also: } b = 22c - 3a = 22 - 12 = 10$$

