

If $a \neq 1$ and ${}^a\sqrt{10000a} = 10a$ then
find a

$$\begin{aligned} & \begin{matrix} 4 & 3 & 2 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{matrix} a \\ & = 0 \times a^0 \\ & \quad + 0 \times a^1 \\ & \quad + 0 \times a^2 \\ & \quad + 0 \times a^3 \\ & \quad + 1 \times a^4 \end{aligned}$$

$$\begin{aligned} \text{So } {}^a\sqrt{10000a} &= {}^a\sqrt{a^4} \\ &= a^{4/a} \end{aligned}$$

$$\begin{aligned} \text{and } 10a &= 0 \times a^0 \\ & \quad + 1 \times a^1 \end{aligned}$$

So we know that

$$a^{4/a} = a$$

$$\Rightarrow a^{\frac{4}{a}-1} = 1$$

and hence $\frac{4}{a} - 1 = 0$

$$\Rightarrow \frac{4}{a} = 1$$

$$\Rightarrow a = 4$$

Let's check: $10_4 = 4$

$$\begin{aligned} \sqrt[4]{10000_4} &= \sqrt[4]{4^4} \\ &= 4 \end{aligned}$$

as expected.