

If a, b are such that

$$\frac{a+b}{a} = \frac{b}{a+b}$$

then a and b cannot both be real.

$$\frac{a+b}{a} = \frac{b}{a+b}$$

$$\Rightarrow (a+b)^2 = ab$$

$$\Rightarrow a^2 + b^2 + 2ab = ab$$

$$\Rightarrow a^2 + b^2 + ab = 0 \quad (*)$$

Consider as a quadratic in a :

$$a^2 + ab + b^2 = 0$$

$$\Rightarrow a = \frac{-b \pm \sqrt{a^2 - 4b^2}}{2}$$

If a real then we need $a^2 \geq 4b^2$

Similarly, by considering $(*)$ as a quadratic in b we need $b^2 \geq 4a^2$

This leads to a contradiction if a & b both real:

$$\begin{aligned} a^2 &\geq 4b^2 \\ &\geq 4(4a^2) \\ &= 16a^2 \end{aligned}$$

Hence at least one of a and b must not be real.