

There are integers $a, b, c > 1$ s.t.

$$\sqrt[a]{N \sqrt[b]{N \sqrt[c]{N}}} = \sqrt[36]{N^{25}}$$

For all $N > 1$

What is B ?

$$\begin{aligned} \text{LHS} &= \left(N \sqrt[b]{N \sqrt[c]{N}} \right)^{\frac{1}{a}} \\ &= N \left(N \sqrt[c]{N} \right)^{\frac{1}{b}} \end{aligned}$$

$$= \left(N \left(N N^{\frac{1}{2}} \right)^{\frac{1}{b}} \right)^{\frac{1}{a}}$$

$$= \left(N \left(N^{1+\frac{1}{2}} \right)^{\frac{1}{b}} \right)^{\frac{1}{a}}$$

$$= \left(N \left(N^{\frac{c+1}{2}} \right)^{\frac{1}{b}} \right)^{\frac{1}{a}}$$

$$= \left(N N^{\frac{c+1}{bc}} \right)^{\frac{1}{a}}$$

$$= \left(N^{1 + \frac{c+1}{bc}} \right)^{\frac{1}{a}}$$

$$= \left(N^{\frac{bc+c+1}{bc}} \right)^{\frac{1}{a}}$$

$$= N^{\frac{bc+c+1}{abc}}$$

$$\text{RHS} = N^{\frac{25}{36}}$$

} equate

$$\frac{bc+c+1}{abc} = \frac{25}{36}$$

$$c=6 \quad b=3 \quad a=2$$

Trial &
error

$$18 + 6 + 1 = 18 + 7 = 25$$

$$abc = 6 \times 6 = 36$$

This works. So $b=3$

