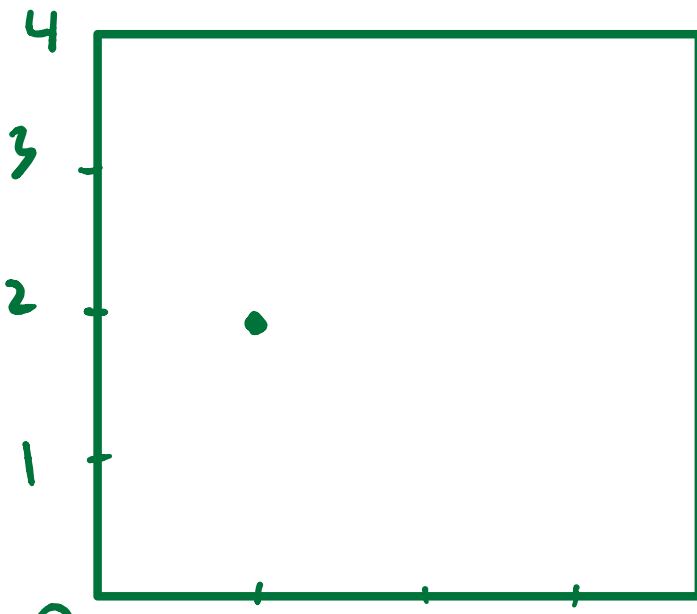


A frog sitting at $(1, 2)$ begins a sequence of jumps. Each parallel to the x or y axis and has length 1. Direction chosen independently at random. Sequence ends when frog reaches a side of the square with vertices $(0, 0)$, $(0, 4)$, $(4, 4)$, $(4, 0)$. What is probability that sequence of jumps ends on a vertical side of the square?



0 1 2 3 4

Frog goes left \rightarrow immediately hits vertical wall

$$P(\text{Frog goes left}) = \frac{1}{4}$$

Frog goes right \rightarrow is now at (2,2) and by symmetry (Frog equidistant from all sides of square), $P(\text{hitting vertical side}) = \frac{1}{2}$

$$P(\text{Frog goes right \& hits vertical wall}) = \frac{1}{4} \times \frac{1}{2} = \frac{1}{8}$$

Frog goes up or down \rightarrow hits diagonal line of symmetry

Equidistant to pair of two closer sides and also pair of two further sides

$$\begin{aligned} P(\text{Frog goes up or down \& hits vertical wall}) &= \frac{1}{2} \times \frac{1}{2} \\ &= \frac{1}{4} \end{aligned}$$

Combine all the cases:

$$\begin{aligned} P(\text{hitting vertical wall}) &= \frac{1}{4} + \frac{1}{8} + \frac{1}{4} \\ &= \frac{1}{2} + \frac{1}{8} \\ &= \frac{5}{8} \end{aligned}$$

