Prove that $1^{99} + 2^{99} + 3^{99} + 4^{99} + 5^{99}$ is divisible by 5.

Clearly 5^{99} is divisible by 5.

We have $1^{99} = 1$.

Now, consider the pattern in the units digit of powers of 2, 3 and 4:

$2^1 = 2$
$2^2 = 4$
$2^3 = 8$
$2^4 \equiv 6$
$2^5 \equiv 2$
$\vdots$
$2^{99} \equiv 8$ since $99 \equiv 3 \mod 4$

$3^1 = 3$
$3^2 = 9$
$3^3 \equiv 7$
$3^4 \equiv 1$
$3^5 \equiv 3$
$\vdots$
$3^{99} \equiv 7$ since $99 \equiv 3 \mod 4$

$4^1 = 4$
$4^2 \equiv 6$
$4^3 \equiv 4$
$4^4 \equiv 6$
$\vdots$
$4^{99} \equiv 4$ since $99 \equiv 1 \mod 2$

Hence $1^{99} + \cdots + 4^{99} \equiv 1 + 8 + 7 + 4 \mod 10$

$\equiv 0 \mod 10$

$\equiv 0 \mod 5$

and so $1^{99} + \cdots + 5^{99}$ is divisible by 5.