How many ordered pairs of integers (x, y) satisfy the equation

$$x^{2020} + y^2 = 2y \ ?$$

A) 1 B) 2 C) 3 D) 4 E) Infinitely many

Rearrange

$$y^2 - 2y + x^{2020} = 0$$

If $x = \pm 1$ then $x^{2020} = 1$ and so

$$y^2 - 2y + 1 = 0$$

$$\Rightarrow (y-1)^2 = 0$$

$$\Rightarrow y = 1$$

So we have at least two ordered pairs $(-1, 1)$ & $(1, 1)$.

If $x = 0$ then

$$y^2 - 2y = 0$$

$$\Rightarrow y(y-2) = 0$$

$\Rightarrow \quad y=0 \quad$ or $\quad y=2$

So we have two more ordered pairs, $(0,0)$ and $(0,2)$, for a total of at least four.

Consider the quadratic

$$y^2 - 2y + x^{2020} = 0$$

We have

$$y = \frac{2 \pm \sqrt{4 - 4x^{2020}}}{2}$$

We need $4 - 4x^{2020} \geq 0$

$$\Rightarrow 1 \geq x^{2020}$$

But we know that $x^{2020} = (x^{1010})^2$
≥ 0

So we must have $x = -1, 0, 1$ as

The only possibilities, which we have already examined. So the answer is D

Another solution method:

$$x^{2020} = 2y - y^2$$
$$= (2-y)y$$

Since LHS ≥ 0 $\forall x$ then $y(2-y) \geq 0$

$\Rightarrow y = 0, 1, 2$ (check all those values)