Show that there are no integers a, b, c for which $a^2 + b^2 - 8c = 6$

$a^2 + (b^2 - 8c - 6) = 0$ is a quadratic in a

$$\Rightarrow a = \frac{-0 \pm \sqrt{0^2 - 4(b^2 - 8c - 6)}}{2}$$

$$= \frac{\pm 2\sqrt{b^2 - 8c - 6}}{2}$$

$$= \pm \sqrt{b^2 - 8c - 6}$$

We know that a is an integer, so

$b^2 - 8c - 6$ must be a perfect square

$n \bmod 4$	$n^2 \bmod 4$
0	0
1	1
2	0
3	1

Now, $8c \equiv 0 \bmod 4$ for any value of c and $6 \equiv 2 \bmod 4$.

If $b^2 \equiv 0 \bmod 4$ then $b^2 - 8c - 6 \equiv 2 \bmod 4$ and therefore camot be a square

If $b^2 \equiv 1 \bmod 4$ then $b^2 - 8c - 6 \equiv 3 \bmod 4$ and therefore camot be a square.

Hence it is impossible to find integers a, b, c for which $a^2 + b^2 - 8c = 6$