

$$\text{Let } a_0 = 2$$

$$a_1 = 5$$

$$a_2 = 8$$

For $n > 2$ define a_n to be the remainder

when $4(a_{n-1} + a_{n-2} + a_{n-3})$ is divided by 11.

$$\text{Find } a_{2018} \cdot a_{2020} \cdot a_{2022}$$

We have $a_n \equiv 4(a_{n-1} + a_{n-2} + a_{n-3}) \pmod{11}$

$$\text{So } a_3 = 4(2 + 5 + 8)$$

$$= 4 \times 15$$

$$= 60$$

$$= 5$$

$$a_4 = 4(5 + 8 + 5)$$

$$= 4(18)$$

$$= 72$$

$$= 6$$

a_0	2	a_{10}	2
a_1	5	a_{11}	5
a_2	8	a_{12}	8
a_3	5		
a_4	6		
a_5	10		
a_6	7		
a_7	4		
a_8	7		
a_9	6		

$$a_5 = 4(6+5+8)$$

$$= 4(19)$$

$$= 76$$

$$= 10$$

$$a_6 = 4(10+6+5)$$

$$= 84$$

$$= 7$$

$$a_7 = 4(10+6+7)$$

$$= 92$$

$$= 4$$

$$a_8 = 4(10+7+4)$$

$$= 84$$

$$= 7$$

$$a_9 = 4(7+4+7)$$

$$= 6$$

$$a_{10} = 4(6+7+4)$$

$$= 68$$

$$= 2$$

$$a_{11} = 4(2+6+7)$$

$$= 60$$

$$= 5$$

$$a_{12} = 4(5+2+6)$$

$$= 52$$

$$= 8$$

And we've returned
to the start, thereby
identifying a
repeating pattern

$$a_0 \rightarrow a_9$$

$$a_{10} \rightarrow a_{19}$$

$$a_{20} \rightarrow a_{29}$$

etc

$$\text{So } a_{2020} = a_0 = 2$$

$$a_{2022} = a_2 = 8$$

$$a_{2018} = a_8 = 7$$

and therefore the required product is

$$7 \times 2 \times 8 = 112$$