

Median of these 4040 numbers?

$1, 2, 3, \dots, 2020, 1^2, 2^2, 3^2, \dots, 2020^2$

$$x^2 = 2020$$

Look for square numbers near 2020

$$2500 = 50 \times 50$$

$$1600 = 40 \times 40$$

40

5

40	1600	200
5	200	25

$$2025 = 45^2$$

So if we order the list, we obtain

$1, 2, 3, \dots, 2020, 45^2, \dots, 2020^2$

$2020 + 44$
numbers

= 2064 numbers

$2020 - 44$
numbers

= 1976 numbers

$$\begin{array}{r} 12064 \\ 1976 \\ \hline 4040 \end{array}$$

We want to find the average of the 2020th and 2021st numbers to obtain the median

$$\begin{aligned}44^2 &= (40+4)^2 \\ &= 1600 + 16 + 2 \times 4 \times 40 \\ &= 1600 + 16 + 320 \\ &= 1920 + 16 \\ &= 1936\end{aligned}$$

Let's just consider the first block of numbers:

$$\underbrace{1, 2, 3, \dots, 1936, 44^2, 1937, \dots, 2020}_{1936 + 44 \text{ numbers}} \quad \begin{array}{c} \uparrow \\ 2064^{\text{th}} \\ \text{number} \end{array}$$

= 1980 numbers

The 2020th number is $1936 + 40 = 1976$

The 2021st number is $1936 + 41 = 1977$

\therefore The median is 1976.5