Integers a, b, c, d (not necessarily distinct) are chosen at random from 0 to 2007 inclusive. What is the probability that $ad - bc$ is even?

$ad - bc$ even $\Rightarrow$ ad, bc both odd

OR

ad, bc both even

$\Rightarrow$ a, b, c, d all odd

OR

at least one of a, d and at least one of b, c are even

In the range $\{0, \ldots, 2007\}$ there are an equal number of odd and even values.

So $P(a, b, c, d \text{ all odd}) = \left(\frac{1}{2}\right)^4 = \frac{1}{16}$

and

P(at least one of a,d even) =

P(a even) + P(d even) − P(both even)

$= \frac{1}{2} + \frac{1}{2} - \frac{1}{4}$

$= \frac{3}{4}$

Similarly, P(at least one of b, c even) = $\frac{3}{4}$

So P(at least one of a, d and at least one of b, c even)

$= \frac{3}{4} \times \frac{3}{4}$

$= \frac{9}{16}$

So P(ad−bc even) $= \frac{1}{16} + \frac{9}{16}$

$= \frac{10}{16}$

$= \boxed{\frac{5}{8}}$

Easier method:

$ad-bc$ is even when ad and bc are either both odd or both even.

$$P(ad \text{ odd}) = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$$

So $$P(ad \text{ even}) = 1 - \frac{1}{4} = \frac{3}{4}$$

$$\therefore P(ad-bc \text{ even}) = \left(\frac{1}{4}\right)^2 + \left(\frac{3}{4}\right)^2 = \frac{5}{8}$$