Cts random variable X has pdf

$$f(x) = \begin{cases} a + bx^2 & 0 \leq x \leq 1 \\ 0 & o/w \end{cases}$$

IF $E(X) = \frac{3}{5}$, determine a & b

$$E(X) = \int_{-\infty}^{\infty} x f(x) \, dx$$

$$= \int_0^1 x(a + bx^2) \, dx$$

$$= \int_0^1 ax + bx^3 \, dx$$

$$= \left[\frac{ax^2}{2} + \frac{bx^4}{4} \right]_0^1$$

$$= \frac{a}{2} + \frac{b}{4}$$

So $\frac{a}{2} + \frac{b}{4} = \frac{3}{5}$

Also note that

$$\int_{-\infty}^{\infty} f(x) \quad dx = 1$$

$$\Rightarrow \int_0^1 a+bx^2 \; dx = 1$$

$$\Rightarrow \left[ax + \frac{bx^3}{3}\right]_0^1 = 1$$

$$\Rightarrow a + \frac{b}{3} = 1$$

So we have simultaneous eqns:

$$a = 1 - \frac{b}{3}$$

$$\Rightarrow \frac{a}{2} = \frac{1}{2} - \frac{b}{6}$$

$$\Rightarrow \frac{1}{2} - \frac{b}{6} + \frac{b}{4} = \frac{3}{5}$$

$$\Rightarrow \frac{b}{12} = \frac{1}{10}$$

$$\frac{1}{4} - \frac{1}{6}$$

$$= \frac{3}{12} - \frac{2}{12}$$

$$\frac{3}{5} - \frac{1}{2}$$

$$= \frac{6}{10} - \frac{5}{10}$$

$$\Rightarrow \quad b = \frac{12}{10} = \frac{6}{5}$$

and $\quad a = 1 - \dfrac{b}{3}$

$$= 1 - \frac{6}{15}$$

$$= \frac{9}{15}$$