

Consider the set $\{1, \dots, 34\}$

How many ways are there to choose (without regard to order) three numbers whose sum is divisible by 3?

We have the following cases:

- ① all numbers $\equiv 0$
- ② all numbers $\equiv 1$
- ③ all numbers $\equiv 2$
- ④ one number $\equiv 0$, one number $\equiv 1$, one number $\equiv 2$

In the set there are 11 numbers $\equiv 0$, 11 numbers $\equiv 1$ and 10 numbers $\equiv 2$.

Case 1 $11C_3 = 165$ ways

Case 2 $11C_3 = 165$ ways

Case 3 $10C_3 = 120$ ways

Case 4 $11 \times 11 \times 10 = 1210$ ways

So there are

$$165 + 165 + 120 + 1210 = 1660$$

Successful choices.

There are ${}_{34}C_3 = 5984$ possible

ways to choose three numbers from the set.

So the probability is $\frac{1660}{5984} = \frac{415}{1496} \approx 0.28$