

If $234_{b+1} - 234_{b-1} = 70_{10}$, then what is 234_b in base 10?

We have

$$2(b+1)^2 + 3(b+1) + 4 - [2(b-1)^2 + 3(b-1) + 4] = 70$$

Expand:

$$2(b^2 + 2b + 1) + 3b + 3 + 4 - [2(b^2 - 2b + 1) + 3b - 3 + 4] = 70$$

Simplify:

$$2b^2 + 4b + 2 + 3b + 7 - [2b^2 - 4b + 2 + 3b + 1] = 70$$

$$\Rightarrow 2b^2 + 7b + 9 - [2b^2 - b + 3] = 70$$

$$\Rightarrow 8b + 6 = 70$$

$$\Rightarrow 8b = 64$$

$$\Rightarrow b = 8$$

$$\text{So } 234_b = 2 \times 64 + 3 \times 8 + 4$$

$$= 128 + 24 + 4$$

$$= 156$$

The number x is 111 when written in base b , but 212 when written in base $b-2$. What is x when written in base 10?

$$x = b^2 + b + 1 \quad (\text{base } b)$$

$$= 2(b-2)^2 + 1(b-2) + 2 \quad (\text{base } b-2)$$

So we have

$$b^2 + b + 1 = 2(b^2 - 4b + 4) + b - 2 + 2$$

$$= 2b^2 - 8b + 8 + b$$

$$= 2b^2 - 7b + 8$$

$$\Rightarrow b^2 - 8b + 7 = 0$$

$$\Rightarrow (b-7)(b-1) = 0$$

$$\Rightarrow b=7 \text{ or } b=1$$

reject since $b-2 > 2$ (it uses the number 2 in its expression of x)

$$\text{Hence } x_{10} = 49 + 7 + 1 = \boxed{57}$$