

A and B are consecutive positive integers.

In the equation below, A, B and A+B represent number bases:

$$132_A + 43_B = 69_{A+B}$$

What is A+B?

Case 1 $B = A + 1$

$$1 \times A^2 + 3 \times A + 2 + 4(A+1) + 3 = 6(2A+1) + 9$$

$$\Rightarrow A^2 + 3A + 5 + 4A + 4 = 12A + 6 + 9$$

$$\Rightarrow A^2 - 5A - 6 = 0$$

$$\Rightarrow (A-6)(A+1) = 0$$

$$\Rightarrow A = 6 \text{ or } A = -1$$

$$\Rightarrow A = 6 \text{ (must be positive) and } B = 7$$

$$\text{So } A+B = 6+7 = 13$$

Case 2 $B = A - 1$

$$1 \times A^2 + 3 \times A + 2 + 4(A-1) + 3 = 6(2A-1) + 9$$

$$\Rightarrow A^2 + 3A + 5 + 4A - 4 = 12A - 6 + 9$$

$$\Rightarrow A^2 - 5A - 2 = 0$$

$$\Rightarrow A = \frac{5 \pm \sqrt{25 - 4(-2)}}{2} = \frac{5 \pm \sqrt{33}}{2}$$

Not possible - we need A to be an integer

So the answer is $A+B = 13$