$$(n+1)! + (n+2)! = 440 \cdot n!$$

$(n \in \mathbb{Z}^+)$

What is the sum of the digits of n?

Required knowledge: - Factorials
- Solving quadratics

Rewrite the equation:

$$(n+1) \cdot n! + (n+2)(n+1) \cdot n! = 440 \cdot n!$$

(Why? Simplifies the algebra by allowing us to get rid of the factorials)

Now divide through by $n!$:

$$n+1 + (n+2)(n+1) = 440$$

Now expand:

$$n+1 + n^2 + 3n + 2 = 440$$

and rearrange:

$$n^2 + 4n - 437 = 0$$

(Why? Because we have put the equation in a familiar form... a quadratic!)

Now we can apply the quadratic Formula:

$$n = \frac{-4 \pm \sqrt{16 - 4 \times (-437)}}{2}$$

$$= \frac{-4 \pm 42}{2}$$

$$= 19, -23$$

But in the question, we were told that n is positive, so we reject the negative solution.

Hence $n = 19$, and so the sum of its digits is $1 + 9 = \boxed{10}$