

What is the largest integer n for which 5^n is a factor of the sum

$$98! + 99! + 100! \quad ?$$

We have

$$\begin{aligned} 98! + 99! + 100! &= 98! [1 + 99 + 99 \times 100] \\ &= 98! [100 + 99 \times 100] \\ &= 98! \times 100^2 \end{aligned}$$

Now, we have

$$\begin{aligned} 100^2 &= 10^4 \\ &= (2 \times 5)^4 \end{aligned}$$

hence this part of the product contributes 4 possible fives.

We must now consider the $98!$. For this we will list all factors of five less than 98. Since this is a fairly short list and therefore quite manageable by brute force.

Factor	# 5's contributed
5	1
10	1
15	1
20	1
25	2
30	1
35	1
40	1
45	1
50	2
55	1
60	1
65	1
70	1
75	2
80	1
85	1
90	1
95	1

$$\text{Total} = 22$$

Add this to our previous four to get the answer $n = 26$