For how many integer values of n is

$$4000 \times \left(\frac{2}{5}\right)^n$$

an integer?

First note that we must consider both positive, zero and negative n.

Now, we factorise 4000:

$$\begin{aligned} 4000 &= 4 \times 10^3 \\ &= 2^2 \times (2 \times 5)^3 \\ &= 2^5 \times 5^3 \end{aligned}$$

So $\left(\frac{2}{5}\right)^n$ must have no more than five 2's or three 5's on the denominator.

Hence $n = 3, 2, 1$ all work (based on the 5's condition)

When $n=0$ we have $\left(\frac{2}{5}\right)^n = 1$ so this also works.

Finally, $n = -1, -2, -3, -4, -5$ all work

(based on the 2's condition)

Therefore there are

$$\underbrace{5}_{\substack{\text{negative} \\ n}} + \underbrace{1}_{n=0} + \underbrace{3}_{\substack{\text{positive} \\ n}} = \boxed{9}$$

values of n which satisfy the requirements.