

Find the remainder when

$$9 \times 99 \times 999 \times \dots \times \underbrace{9 \dots 9}_{999 \text{ nines}}$$

is divided by 1000

We want to find that product mod 1000.

We have $999 \equiv -1 \pmod{1000}$

$$9999 = 9 \times 1000 + 999$$

$$\equiv -1 \pmod{1000}$$

and similarly, every higher number in the product is in the congruence class of -1 .

So the product

$$\underbrace{999}_{3 \text{ nines}} \times \dots \times \underbrace{9 \dots 9}_{999 \text{ nines}}$$

$$\equiv (-1) \times \dots \times (-1) \pmod{1000}$$

$\underbrace{\hspace{10em}}$
 $999 - 3 + 1 = 997$ numbers in this product

$$\equiv -1 \pmod{1000}$$

$$\therefore 9 \times 99 \times \dots \times 9 \dots 9 \equiv 9 \times 99 \times (-1) \pmod{1000}$$

$$\equiv -891 \pmod{1000}$$

$$\equiv 1000 - 891 \pmod{1000}$$

$$\equiv 109 \pmod{1000}$$